



TITLE:

Three-Particle Relativistic Kinematics (Commemoration Issue Dedicated to Professor Sakae Shimizu on the Occasion of his Retirement)

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Three-Particle Relativistic Kinematics

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For reactions leading to three particles in the final state, five independent variables are assigned to the polar and azimuthal angles of two particles and the kinetic energy of one of them. Formulas are given for calculations of other variables from these five. Formulas are also given for transformations of polar and azimuthal angles from the laboratory system to the rest frames of two-particle systems. A FORTRAN programme using these formulas is included in an appendix.

KEY WORDS Three-particle kinematics / Relativistic formulas /
FORTRAN programme /

I. INTRODUCTION

For three-particle reactions of the form $a+b \rightarrow 1+2+3$, five independent variables are required to define the final state completely. In experiments of two-particle correlation measurements, two detectors are set at angles (θ_1, ϕ_1) and (θ_2, ϕ_2) respectively and kinetic energies T_1 and T_2 of particles 1 and 2 are measured in coincidence. Therefore, quantities T_1 , θ_1 , ϕ_1 , θ_2 , and ϕ_2 are taken as five independent variables and differential cross sections $d^5\sigma/dT_1 d\Omega_1 d\Omega_2$ are measured as functions of these variables. Kinetic energy T_3 , an extra variable, can be used to identify true events of the reaction.

Coincident energy spectra are characterized by relative kinetic energies between two final-state particles. These energies are calculated from the five independent variables. For the purpose of doing it, nonrelativistic formulas are summarized in Ref. 1. Since two-particle correlation experiments are performed frequently at intermediate incident energies, relativistic calculations are done in this report. Moreover the following matters are examined. First, quasi-free scattering proceeds in two ways. One is that the target b consists of two particles and one of them collides with the projectile a with leaving the rest particle as a spectator. The other is that the projectile a consists of two particles and one of them collides with the target b . In the former, quasi-free scattering occurs near the energy corresponding to the spectator kinetic energy of zero in the target system and in the latter, of zero in the beam system. These two processes of quasi-free scattering are expected to occur especially when particles a and b are identical.²⁾ Then, kinetic energies T_i of particles i are calculated in both the target and

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beam systems. Secondly, in angular-correlation experiments, differential cross sections $d^5\sigma/dT_1 d\Omega_1 d\Omega_2$ are measured as functions of angles (θ_2, ϕ_2) of particle 2 if one assigns particle 1 to the particle emitted from the production process of the (23) system and particle 2 to the succeeding decaying one. Angular correlation functions are obtained through the transformation of differential cross sections to the ones in the rest frame of (23) system. Several times, according to physical or experimental requirements, detector 2 is placed off the reaction plane which is defined by the direction of detector 1 and that of incident beam. Then transformation equations for the azimuthal angle ϕ_2 as well as for the polar angle θ_2 are required.

II. CALCULATION FORMULAS

1. Reference Systems

Four types of reference system³⁾ are used to describe kinetic motions of particles, that is, laboratory system (LS), overall center-of-momentum system (CMS), beam system (BS) and rest frames of two-particle system (R12, R23, and R31). Target b is assumed to rest in the LS and therefore the target system is identical with the LS. The CMS quantities are denoted by an asterisk and the Rij quantities by an index Rij. An index L for the LS quantities is omitted for simplicity.

The z-axis of the LS is defined as the momentum of projectile and the z-axis in the CMS is defined as the direction parallel to the LS one. The zx-plane ($\phi=0$) can be defined arbitrary.

2. Kinetic Energies

Quantities to describe the motion of particle i are as follows: the total energy E_i , the momentum \mathbf{P}_i , the rest mass m_i , the kinetic energy T_i , and the velocity \mathbf{v}_i . The momentum vector is expressed in terms of its absolute value P_i , the polar angle θ_i , and the azimuthal angle ϕ_i in the polar coordinate system. Among these quantities exist the following relations.

$$E_i^2 = P_i^2 + m_i^2, \quad (1)$$

$$E_i = T_i + m_i, \quad (2)$$

$$P_i^2 = T_i^2 + 2m_i T_i, \quad (3)$$

$$\mathbf{v}_i = \mathbf{P}_i / E_i. \quad (4)$$

The similar relations hold for a two-particle system consisting of particles i and j with definition of its energy E_{ij} , momentum \mathbf{P}_{ij} and invariant mass M_{ij} as

$$E_{ij} = E_i + E_j, \quad (5)$$

$$\mathbf{P}_{ij} = \mathbf{P}_i + \mathbf{P}_j, \quad (6)$$

$$M_{ij}^2 = (E_i + E_j)^2 - (\mathbf{P}_i + \mathbf{P}_j)^2. \quad (7)$$

For the total system its energy E_0 , momentum \mathbf{P}_0 and invariant mass M_0 are calculated from the quantities of the initial state with assigning the particle a to the projectile and the particle b to the target. They are written as

$$E_0 = E_a + E_b = T_a + m_a + m_b, \quad (8)$$

$$\mathbf{P}_0 = \mathbf{P}_a + \mathbf{P}_b = \mathbf{P}_a, \quad (9)$$

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$$P_0 = P_a = (T_a^2 + 2m_a T_a)^{1/2}, \quad (10)$$

$$M_0^2 = E_0^2 - P_0^2 = (m_a + m_b)^2 + 2m_b T_a. \quad (11)$$

If, for the particle 1, the kinetic energy T_1 and the angles (θ_1, ϕ_1) are known in the LS, the (23) system consisting of particles 2 and 3 is uniquely determined owing to the energy-momentum conservation equations

$$E_0 - E_1 = E_2 + E_3, \quad (12)$$

$$\mathbf{P}_0 - \mathbf{P}_1 = \mathbf{P}_2 + \mathbf{P}_3. \quad (13)$$

Then kinematics for two-particle reactions⁴⁾ can be applied to solve P_2 as a function of the angles (θ_2, ϕ_2) . The calculations are performed as follows. First, in the R23, the momentum and the energy of particle 2 are independent of the angles and are expressed in terms of the invariant mass M_{23} :

$$P_2^{R23} = \lambda^{1/2}(M_{23}^2, m_2^2, m_3^2) / (2M_{23}), \quad (14)$$

$$E_2^{R23} = (M_{23}^2 + m_2^2 - m_3^2) / (2M_{23}), \quad (15)$$

where

$$\begin{aligned} M_{23}^2 &= (E_2 + E_3)^2 - (\mathbf{P}_2 + \mathbf{P}_3)^2 \\ &= (E_0 - E_1)^2 - (\mathbf{P}_0 - \mathbf{P}_1)^2 \\ &= M_0^2 + m_1^2 - 2E_0 E_1 + 2P_0 P_1 \cos\theta_1 \end{aligned} \quad (16)$$

and the function $\lambda(x, y, z)$ is defined as

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx. \quad (17)$$

Secondly, the momentum of particle 2 in the LS is calculated from the momentum and energy in the R23 through the Lorentz transformation between these systems. The transformation is done along the z' -axis which is defined as \mathbf{P}_{23} in the LS. The velocity \mathbf{v}_{23} of the R23 in the LS and the associated Lorentz factor γ_{23} are given by

$$\mathbf{v}_{23} = \mathbf{P}_{23} / E_{23}, \quad (18)$$

$$\gamma_{23} = E_{23} / M_{23}, \quad (19)$$

$$\gamma_{23} v_{23} = P_{23} / M_{23}, \quad (20)$$

where

$$E_{23} = E_0 - E_1, \quad (21)$$

$$P_{23} = |\mathbf{P}_0 - \mathbf{P}_1| = (P_0^2 + P_1^2 - 2P_0 P_1 \cos\theta_1)^{1/2}. \quad (22)$$

The angle θ_{23} of (23) system with respect to the z -axis is given through the momentum conservation, Eq. (13), as follows:

$$\sin\theta_{23} = P_1 \sin\theta_1 / P_{23}, \quad (23)$$

$$\cos\theta_{23} = (P_0 - P_1 \cos\theta_1) / P_{23}. \quad (24)$$

Now, the Lorentz transformation equation solved for P_2 is written as

$$\begin{aligned} E_2^{R23} &= -\gamma_{23} v_{23} P_2 \cos\theta'_2 + \gamma_{23} E_2 \\ &= -\gamma_{23} v_{23} P_2 \cos\theta'_2 + \gamma_{23} (P_2^2 + m_2^2)^{1/2}. \end{aligned} \quad (25)$$

The angle θ'_2 is measured with respect to the z' -axis and $\cos\theta'_2$ is expressed by

$$\begin{aligned} \cos\theta'_2 &= \cos\theta_2 \cos\theta_{23} - \sin\theta_2 \cos(\phi_2 - \phi_1) \sin\theta_{23}, \\ &= (P_0 \cos\theta_2 - P_1 \cos\theta_{1-2}) / P_{23}, \end{aligned} \quad (26)$$

where

$$\mathbf{P}_3 = \mathbf{P}_0 - \mathbf{P}_1 - \mathbf{P}_2. \quad (38)$$

From the latter equations, P_3 is derived as follows.

$$P_3 = (P_0^2 + P_1^2 + P_2^2 - 2P_0P_1\cos\theta_1 - 2P_0P_2\cos\theta_2 + 2P_1P_2\cos\theta_{1-2})^{1/2}. \quad (39)$$

The kinetic energy T_1 of particle 1 is given by

$$T_1 = E_1 - m_1. \quad (40)$$

The kinetic energies of the relative motions between the particles j and k and between the particle i and the (jk) system are given by

$$T_{j-k} = M_{jk} - m_j - m_k \quad (41)$$

and

$$T_{i-jk} = M_0 - m_i - M_{jk} \quad (42)$$

respectively. M_{jk} are obtained by cyclic permutations of indexes in Eq. (16).

Once the energy E_1 is obtained in the LS, the energy E_1^B in the BS can be calculated through the Lorentz transformation between these systems with $\gamma_a = E_a/m_a$ and $\gamma_a v_a = P_a/m_a$ and is given by

$$E_1^B = (E_a E_1 - P_a P_1 \cos\theta_1) / m_a. \quad (43)$$

3. Polar and Azimuthal Angles

In order to evaluate an azimuthal angle ϕ , both values of $\sin\phi$ and $\cos\phi$ are needed and on the contrary a polar angle θ is uniquely evaluated from a value of $\cos\theta$. However, in practical calculations with a computer, it is frequently convenient to use both values $\sin\theta$ and $\cos\theta$ for evaluating θ . In the following both formulas for $\sin\theta$ and $\cos\theta$ are given.

The LS angles (θ_3, ϕ_3) of particle 3 are calculated from the quantities for the particles 1 and 2 through the momentum conservation, Eq. (38), and are given by

$$\sin\theta_3 = (P_1^2 \sin^2\theta_1 + P_2^2 \sin^2\theta_2 + 2P_1P_2 \sin\theta_1 \sin\theta_2 \cos(\phi_2 - \phi_1))^{1/2} / P_3, \quad (44)$$

$$\cos\theta_3 = (P_0 - P_1 \cos\theta_1 - P_2 \cos\theta_2) / P_3, \quad (45)$$

$$\sin\phi_3 = (-P_1 \sin\theta_1 \sin\phi_1 - P_2 \sin\theta_2 \sin\phi_2) / (P_3 \sin\theta_3), \quad (46)$$

$$\cos\phi_3 = (-P_1 \sin\theta_1 \cos\phi_1 - P_2 \sin\theta_2 \cos\phi_2) / (P_3 \sin\theta_3). \quad (47)$$

Practically the factor $P_3 \sin\theta_3$ in Eqs. (46) and (47) is not needed because ϕ_3 is evaluated through the ratio $\tan\phi_3 = \sin\phi_3 / \cos\phi_3$ associated with an examination of the sign of $\sin\phi_3$ and $\cos\phi_3$ ($\sin\theta_3 \geq 0$). For calculation of θ_3 through Eq. (45) only, the value of P_3 is given by Eq. (39).

The CMS angles (θ_1^*, ϕ_1^*) are calculated from the LS ones through the Lorentz transformation equations with $\gamma_0 = E_0/M_0$ and $\gamma_0 v_0 = P_0/M_0$ which are evaluated with Eqs. (8), (10) and (11), and are given by

$$\sin\theta_1^* = P_1 \sin\theta_1 / P_1^*, \quad (48)$$

$$\cos\theta_1^* = (E_0 P_1 \cos\theta_1 - P_0 E_1) / (M_0 P_1^*), \quad (49)$$

$$\phi_1^* = \phi_1. \quad (50)$$

If one calculates θ_1^* through Eq. (49) only, the value of P_1^* is needed and is given by

$$P_1^* = \lambda^{1/2} (M_{jk}^2, M_0^2, m_1^2) / (2M_0). \quad (51)$$

In the Rjk, the z-axis is defined as the direction opposite to the momentum of particle i, that is, $-\mathbf{P}_i^{\text{Rjk}}$. Then the polar angle θ_j^{Rjk} equals to $\pi - \theta_{i-j}^{\text{Rjk}}$ and is expressed in terms of the invariant mass M_{ij} and the energies and momenta of particles i and j in the Rjk as follows:

$$\cos\theta_j^{\text{Rjk}} = (M_{ij}^2 - m_i^2 - m_j^2 - 2E_i^{\text{Rjk}}E_j^{\text{Rjk}}) / (2P_i^{\text{Rjk}}P_j^{\text{Rjk}}), \quad (52)$$

where

$$E_i^{\text{Rjk}} = (M_0^2 - M_{jk}^2 - m_i^2) / (2M_{jk}), \quad (53)$$

$$P_i^{\text{Rjk}} = \lambda^{1/2}(M_0^2, M_{jk}^2, m_i^2) / (2M_{jk}) \quad (54)$$

and E_j^{Rjk} and P_j^{Rjk} are obtained by cyclic permutations of indexes in Eqs. (14) and (15). The Rjk azimuthal angle ϕ_j^{Rjk} of particle j is calculated from the LS angles through three steps of transformations. The first is the Lorentz transformation between the LS and the CMS along the z-axis. The azimuthal angle is invariant, that is, $\phi_j = \phi_j^*$. The second is a rotation of coordinate system in the CMS which transform the z-axis to the z'' -axis. The latter axis is defined as $\mathbf{P}_{jk}^* = -\mathbf{P}_i^*$ and the rotation angle equals to the angle $\theta_{jk}^* = \pi - \theta_i^*$. By using formulas of spherical trigonometry, ϕ_j^* is transformed to ϕ_j'' through the following equations.

$$\sin\theta_j''\sin\phi_j'' = \sin\theta_j^*\sin(\phi_j^* - \phi_i^*) \quad (55)$$

$$\sin\theta_j''\cos\phi_j'' = \cos\theta_j^*\sin\theta_i^* - \sin\theta_j^*\cos\theta_i^*\cos(\phi_j^* - \phi_i^*). \quad (56)$$

The $z''x''$ -plane ($\phi''=0$) is defined as the plane including the z-axis (Fig. 2). The last is a Lorentz transformation between the CMS and the Rjk along the z'' -axis and the equations are as follows:

$$P_j^{\text{Rjk}}\sin\theta_j^{\text{Rjk}} = P_j^*\sin\theta_j'', \quad (57)$$

$$\phi_j^{\text{Rjk}} = \phi_j''. \quad (58)$$

By substitution of Eqs. (48), (49), (50), (57) and (58) into Eqs. (55) and (56), the following equations are obtained:

$$\sin\phi_j^{\text{Rjk}} = P_j\sin\theta_j\sin(\phi_j - \phi_i) / (P_j^{\text{Rjk}}\sin\theta_j^{\text{Rjk}}), \quad (59)$$

$$\cos\phi_j^{\text{Rjk}} = [(E_0P_j\cos\theta_j - P_0E_j)P_i\sin\theta_i - (E_0P_i\cos\theta_i - P_0E_i)P_j\sin\theta_j\cos(\phi_j - \phi_i)] \\ \times (M_0P_i^*)^{-1}(P_j^{\text{Rjk}}\sin\theta_j^{\text{Rjk}})^{-1}. \quad (60)$$

Concerning to the factor $(P_j^{\text{Rjk}}\sin\theta_j^{\text{Rjk}})$, is noted the same fact as mentioned about Eqs. (46) and (47).

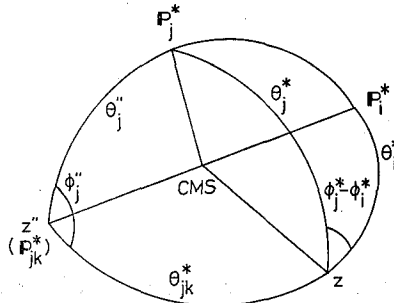


Fig. 2. Angle coordinate in the center-of-momentum system.

4. Phase Space Factor

If the differential cross section is written in a form as follows¹⁾:

$$d^5\sigma/dT_1 d\Omega_1 d\Omega_2 = (2\pi/\hbar v_a) \rho_1(E_1) |M|^2, \quad (61)$$

then the phase space factor $\rho_1(E_1)$ is given by

$$\rho_1(E_1) = (2\pi\hbar)^{-6} P_1 P_2 E_1 E_2 E_3 |E_2 + E_3 - E_2(\mathbf{P}_0 - \mathbf{P}_1) \cdot \mathbf{P}_2 / P_2^2|^{-1}. \quad (62)$$

This equation corresponds to the following definition:

$$\begin{aligned} \rho_1(E_1) dT_1 d\Omega_1 d\Omega_2 &= (2\pi\hbar)^{-6} \int d^3\mathbf{P}_1 d^3\mathbf{P}_2 d^3\mathbf{P}_3 \\ &\times \delta^3(\mathbf{P}_0 - \mathbf{P}_1 - \mathbf{P}_2 - \mathbf{P}_3) \delta(E_0 - E_1 - E_2 - E_3) \end{aligned} \quad (63)$$

with the integrations over the variables P_3 , θ_3 , ϕ_3 , and P_2 . This expression is a non-invariant form and differs from a Lorentz-invariant one by inclusion of a factor of $8E_1 E_2 E_3$. In the R23, the phase space factor is calculated in the form:

$$\rho_1(E_1^{R23}) = (2\pi\hbar)^{-6} P_1^{R23} P_2^{R23} E_1^{R23} E_2^{R23} E_3^{R23} / M_{23}. \quad (64)$$

Then the Jacobian $\partial(T_1, \Omega_1, \Omega_2)/\partial(T_1^{R23}, \Omega_1^{R23}, \Omega_2^{R23})$ for the transformation of the differential cross section $d^5\sigma/dT_1 d\Omega_1 d\Omega_2$ from the LS to the R23 is given by

$$\partial(T_1, \Omega_1, \Omega_2)/\partial(T_1^{R23}, \Omega_1^{R23}, \Omega_2^{R23}) = \rho_1(E_1^{R23})/\rho_1(E_1). \quad (65)$$

III. PROGRAMME

A FORTRAN programme is given in an appendix. The input quantities are the masses of five particles participating in the reaction: m_a , m_b , m_1 , m_2 , and m_3 (in AMU), the kinetic energy of the projectile in the laboratory system: T_a (in MeV) and the polar and azimuthal angles of the particles 1 and 2 in the laboratory system: θ_1 , ϕ_1 , θ_2 , and ϕ_2 (in degrees). In calculations the kinetic energies of particle 1: T_1 (in MeV) are given successively by a starting value plus a increment value multiplied by integers. These values are also given in the input data. In the programme is put a restriction that a given value of T_1 should satisfy a limit $M_{23} - m_2 - m_3 \geq 0$, before the examinations mentioned in the preceding section. The output quantities are represented in a matrix form as used in Ref. 1.

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APPENDIX

SOURCE LIST

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C      THREE BODY RELATIVISTIC KINEMATICS
C
C      P=PROJECTILE
C      T=TARGET
C      I=DETECTED AT (TH1L,PH1L)
C      2=DETECTED AT (TH2L,PH2L)
C      3=UNDTECTED
C
C      L =LAB.
C      C =CM OF TOTAL SYSTEM
C      RIJ=CM OF (IJ)SYSTEM
C      R =REAM
C
C      XM =REST MASS (AMU)
C      T =KINETIC ENERGY (MEV)
C      TH =POLAR ANGLE (DEG.)
C      PH =AZIMUTHAL ANGLE (DEG.)
C      TI-J=RELATIVE ENERGY OF PARTICLES I-J
C      PHOJ=PHASE SPACE (T1L)
C      PHOJ=PHASE SPACE (T1L,T2L)
C      GHOJ=JACOBIAN FOR TRANSFORMATION L-R23
C
0001  COMMON/DIN1/XMP,XM1,XM2,XM3,TPL,TH1L,TH2L,PH1L,PH2L
0002  COMMON/DIN2/START,STEP
0003  DOUBLE PRECISION XMP,YMT,XM1,XM2,XM3
0004  READ(5,1000) XMP
0005  READ(5,1000) XMT
0006  READ(5,1010) TPL
0007  READ(5,1020) START,STEP
0008  10 READ(5,1000) XM1
0009  READ(5,1000) XM2
0010  READ(5,1000) XM3
0011  IF (XM1.EQ.0.0) GO TO 900
0012  20 READ(5,1020) TH1L,PH1L
0013  IF (TH1L.EQ.360.0) GO TO 10
0014  30 READ(5,1020) TH2L,PH2L
0015  IF (TH2L.EQ.360.0) GO TO 20
0016  CALL KINEMA
0017  CALL PRINT
0018  GO TO 30
0019  900 CONTINUE
0020  STOP
0021  1000 FORMAT(D14.8)
0022  1010 FORMAT(F6.2)
0023  1020 FORMAT(F7.2)
0024  END
C
0001  SUBROUTINE KINEMA
0002  COMMON/DIN1/XMP,XMT,XM1,XM2,XM3,TPL,TH1L,TH2L,PH1L,PH2L
0003  COMMON/DIN2/START,STEP
0004  COMMON/DOU1/T1L(300,2),T2L(300,2),T3L(300,2)
X=T2(300),T3(300,2),T2(300,2),TH3L(300,2),PH3L(300,2)
X=TH3C(300),TH2C(300,2),TH3C(300,2),RHO1(300,2),RHO2(300,2)
X=TH2R23(300,2),TH3R31(300,2),TH1R12(300,2)
X=PH2R23(300,2),PH3R31(300,2),PH1R12(300,2)
X=GBCM(300,2)
X=T1R(300),T2R(300,2),T3R(300,2)
COMMON/DOU2/RLV,E0,P0,R50
COMMON/LN1/CD/LN1R,LMAX,LCODE,LCODE
DOUBLE PRECISION XMP,XMT,XM1,XM2,XM3,YNP,YMT,YM1,YM2,YM3,Z1,Z2,Z3
X=ZP,ZT
X=AUMV,ARG,PAI
X=TH1,TH2,TH3,PH1,PH2,PH3,TH1CM,TH2CM,TH3CM
X=E0,P0,S0,H0,H6V
X=P1,P2,P3,E1,E2,E3,E23,P23,V23,G23,COTH2,COTH2P
X=E1R23,E2R23,E3R23,P1R23,P2R23,V2R23,G2R23
X=S23,S31,S12,T1,T2,T3

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X=A,B,D,VH,DVR,RT23
X=THCN,THRCM,PHRCM,ARCTAN,NAMDA,X,Y,Z
X=RI1,RI1CM,FACT,EFACT1,EFACT2
X=S1,CO,RNS,RNB
RANMA(X,Y,Z)=X**2+Y**2+Z**2-2.0*X*Y-2.0*Y*Z-2.0*Z*X
AUMV=931.504
ARG=-1.0
PAI=DAKCOS(ARG)
YMP=XMP*AUMV
YMT=XMT*AUMV
YM1=XM1*AUMV
YM2=XM2*AUMV
YM3=XM3*AUMV
ZP=YMP**2
ZT=YMT**2
Z1=YM1**2
Z2=YM2**2
Z3=YM3**2
TH1=TH1L*PAI/180.0
TH2=TH2L*PAI/180.0
PH1=PH1L*PAI/180.0
PH2=PH2L*PAI/180.0
EQ=YMP*YMT+TPL
PO=DSORT(TPL**2+2.0*YMP*TPL)
SO=E0**2-P0**2
RSC=DSORT(SO)
EP=YMP+TPL
EQ=YMP+YMT-YM1-YM2-YM3
LCODE=0
LCODE=0
LMIN=1
LMAX=300
DD 505 L=1.306
XL=L-1
TI=START+STEP*XL
F1=T1+YMT
10 P1=DSORT(T1**2+2.0*YMT+T1)
S23=S0+Z1-2.0*E0*E1+2.0*P0*P1*DCOS(TH1)
RT23=DSORT(S23)-YMT-YM3
IF (RT23) 60,15,15
15 CONTINUE
E23=E0-E1
P23=DSORT(P0**2+P1**2-2.0*P0*P1*DCOS(TH1))
V23=P23/F23
G23=E23/DSORT(S23)
P1P23=DSORT(RANMA(S0,S23,Z1))/(2.0*DSORT(S23))
P2R23=DSORT(RANMA(S23,Z2,Z3))/(2.0*DSORT(S23))
E1R23=(S0-S23-Z1)/(2.0*DSORT(S23))
E2R23=(S23+Z2-Z1)/(2.0*DSORT(S23))
E3R23=(S23+Z3-Z2)/(2.0*DSORT(S23))
V2R23=P2R23/E2R23
G2R23=E2R23/YM2
VH=V23/V2R23
COTH12=DCOS(TH1)*DCOS(TH2)*DSIN(TH1)*DSIN(TH2)*RCOS(PH2-PH1)
COTH2P=(P0*DCOS(TH2)-P1*COTH12)/P23
D=(G23**2)*(1.0-VF**2)*(VR**2)*(COTH2P**2)+(S23/G2H23)**2
A=G23*(1.0-(V23**2)*(COTH2P**2))
B=VR*COTH2P
DVR=1.0-VH
IF (DVR) 25,40,40
25 IF (COTH2P) 60,30,30
40 LOWCD=0
TIL(L,1)=T1
TIL(L,2)=0.0
GO TO 40
30 IF (D) 60,50,50
50 LOWCD=1
LCODE=1
TIL(L,1)=T1
TIL(L,2)=T1

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0074      GO TO 90
0075      60 CONTINUE
0076      IF (ICODF.EQ.0) GO TO 500
0077      LMAX=L-1
0078      GO TO 600
0079      90 IF (ICODF.EQ.1) GO TO 80
0080      LMIN=L
0081      ICODEF=1
0082      80 K=1
0083      P2=P2H23*(R+DSQRT(D))/A
0084      GO TO 100
0085      95 K=2
0086      P2=P2H23*(R-DSQRT(D))/A
0087      CONTINUE
0088      F2=DSQRT(P2**2/(Z))
0089      P3=DSQRT((P1**2+1**2+P2**2-2.0*P0*P1*DCOS(TH1)
0090      Y=2.0*P0*P2*DCOS(TH1)/2.0*P1*P2*COTH12)
0091      F3=DSQRT(P3**2/(Z))
0092      S31=S0*Z2-2.0*P0*P2*DCOS(TH2)
0093      S12=Z1*Z2*U*E1+E2-2.0*P1*P2*COTH12
0094      T2L(L,K)=E2-YN2
0095      T3L(L,K)=E2-YN3
0096      T23(L,K)=DSQRT(S23)-YN2-YN3
0097      T31(L,K)=DSQRT(S31)-YN3-YM1
0098      T12(L,K)=DSQRT(S12)-YM1-YM2
0099      TA1=Z1*Z1-2.0*P1*P1-2.0*P0*P1*DCOS(TH1)
0100      TA2=Z2*Z2-2.0*P1*P1-2.0*P0*P2*DCOS(TH2)
0101      TA3=Z1*Z2+Z1*Z2*U*(1+1/2*YMT*(E1+E2)-P1*P2*COTH12)
0102      T1R(L,K)=((YMP-YM1)**2-TA1)/(2.0*YMP)
0103      T2R(L,K)=((YMP-YM2)**2-TA2)/(2.0*YMP)
0104      T3R(L,K)=((YMP-YM3)**2-TA3)/(2.0*YMP)
0105      S1=DSQRT((P1**2*DCOS(TH1)+P2**2*DCOS(TH2))**2
0106      Y=2.0*P1*P2*DSIN(TH1)*DSIN(TH2)*DCOS(PH2-PH1)
0107      CU=P0*P1*DCOS(TH1)-P2*DCOS(TH2)
0108      TH3=ARCTAN(S1,CO)
0109      TH31(L,K)=TH3*180.0/PAI
0110      S1E=-P1*DSIN(TH1)*DSIN(PH1)-P2*DSIN(TH2)*DSIN(PH2)
0111      COE=-P1*DSIN(TH1)*DCOS(PH1)-P2*DSIN(TH2)*DCOS(PH2)
0112      PH3=ARCTAN(S1E,COE)
0113      PH31(L,K)=PH3*180.0/PAI
0114      TH1CM=THCM(TH1,S0,E0,P0,E1,P1)
0115      TH2CM=THCM(TH2,S0,E0,P0,E2,P2)
0116      TH3CM=THCM(TH3,S0,E0,P0,E3,P3)
0117      TH1C(L,K)=TH1CM*180.0/PAI
0118      TH2C(L,K)=TH2CM*180.0/PAI
0119      TH3C(L,K)=TH3CM*180.0/PAI
0120      TH2P23(L,K)=THCM(S0,S12,S23,Z1,Z2,Z3)
0121      TH2P12(L,K)=THCM(S0,S31,S12,Z2,Z3,Z1)
0122      TH2P23(L,K)=THCM(TH1CM,TH2CM,PH1,PH2)
0123      PH1P12(L,K)=PHCM(TH2CM,TH3CM,PH2,PH3)
0124      PH1P12(L,K)=PHCM(TH3CM,TH1CM,PH3,PH1)
0125      FACT=1.0
0126      EFAC1=E1*E2*E3
0127      B15=P1*P2*EFAC1
0128      RH1=BNS/PNH
0129      PH01(L,K)=RH1
0130      EFAC2=F1*H23*F2*H23*F3*H23
0131      BNS=P1*H23*H23*EFAC2
0132      RH2=DSQRT(S23)*FACT
0133      PH1RCM=BNS/RH2
0134      GHCV(L,K)=180.0/PAI
0135      IF (LOWCD.EQ.0) GO TO 500
0136      IF (X.LE.2) GO TO 500
0137      GO TO 95
0138      500 CONTINUE
0139      505 CONTINUE
0140      600 RETURN
0141      END

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0001      FUNCTION ARCTAN(X,Y)
0002      DOUBLE PRECISION ARCTAN,X,Y,PAI
0003      DOUBLE PRECISION ARG
0004      ARG=1.0
0005      PAI=0.0
0006      IF (X) 300,301,302
0007      300 IF (Y) 303,303,303
0008      303 ARCTAN=DATAN(Y/X)+PAI
0009      GO TO 390
0010      301 IF (Y) 306,307,308
0011      306 ARCTAN=3.0*PAI/2.0
0012      GO TO 390
0013      307 ARCTAN=999.0*PAI/180.00
0014      GO TO 390
0015      308 ARCTAN=PAI/2.0
0016      GO TO 390
0017      302 IF (Y) 309,310,310
0018      309 ARCTAN=DATAN(Y/X)+2.0*PAI
0019      GO TO 390
0020      310 ARCTAN=DATAN(Y/X)
0021      390 CONTINUE
0022      RETURN
0023      END

0001      FUNCTION THCM(TH1,S0,E0,P0,E1,P1)
0002      DOUBLE PRECISION TH1,S0,E0,P0,E1,P1,BNS,BNB,THCM
0003      DOUBLE PRECISION ARCTAN
0004      BNS=DSQRT(S0)*DSIN(TH1)
0005      BNB=E0*DCOS(TH1)-P0*(E1/P1)
0006      THCM=ARCTAN(BNS,BNB)
0007      RETURN
0008      END

0001      FUNCTION THRCM(S0,S12,S23,Z1,Z2,Z3)
0002      DOUBLE PRECISION THRCM,S0,S12,S23,Z1,Z2,Z3,PAMPA,U,V,W
0003      X=R1+R2+BNS+BNB*X,PAI
0004      DOUBLE PRECISION ARG
0005      RAMPDA(U,V,W)=U**2+V**2+W**2-2.0*U*V-2.0*V*W-2.0*W*U
0006      ARG=1.0
0007      PAI=0.0
0008      R1=RAMPDA(S0,S23,Z1)
0009      R2=RAMPDA(S23,Z2,Z3)
0010      BNS=(S0-S23-Z1)*(S23+Z2-Z3)+2.0*S23*(Z1+Z2-S12)
0011      BNB=DSQRT(W1)*DSQRT(R2)
0012      X=BNS+BNB
0013      THRCM=ARCTAN(X)*180.0/PAI
0014      RETURN
0015      END

0001      FUNCTION PHRCM(TH1CM,TH2CM,PH1,PH2)
0002      DOUBLE PRECISION PHRCM,TH1CM,TH2CM,PH1,PH2,PH12,S1,CO,PAI,ARG
0003      X=ARCTAN
0004      ARG=1.0
0005      PAI=0.0
0006      PH12=PH2-PH1
0007      S1=DSIN(TH2CM)*DSIN(TH12)
0008      CO=DCOS(TH2CM)*DSIN(TH1CM)-DCOS(TH1CM)*DSIN(TH2CM)*DCOS(PH12)
0009      PHRCM=ARCTAN(S1,CO)
0010      PHRCM=PHRCM*180.0/PAI
0011      RETURN
0012      END

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